

Satellite Oceanography: an integrated perspective

help reveal and model unknow unknowns from all available multi-modal satellite ocean remote sensing measurements ?

How to contribute/accelerate new skills

Some Earth Observation Challenges:

Upper vertical motions i.e. 3D dynamics (e.g. including time evolution) of the upper ocean, Mesoscale and submesoscale circulation as key to control the vertical ocean pump and its impact on energy transport and biogeochemical cycles

Climate modelling due to these vast and diverse scales of fluid motions: in the upper ocean, horizontal scales as big as basins and as small as cmmm (capillary-gravity surface waves) contribute non-negligibly to air-sea exchanges and climate, and dynamics of scales of less than 30 km, is characterized by departures from the Earth's rotation constraint, i.e ageostrophic motions and strong impact of wind/wave transient forcings.



Problem: introduction of new regimes for heat, salinity and chemistry distribution which threaten the sustainability of life in the sea and increase the frequency of extreme events !





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Future goals are to contribute/accelerate to new skills/discoveries to help reveal and model unknow unknowns from all available multi-modal satellite ocean remote sensing measurements ?

Computational simulations have a limiting "<u>irreducible imprecision</u>" compared to measured quantities in the turbulent regimes of the upper ocean. This limiting feature explains the observed irreproducibility among different model schemes which are supposed to be solving the same problem. The imprecision of simulations is due to the variety of independent selections of different numerical algorithms, model parameterizations, and representations of couplings among the different processes.

70% Earth's surface covered by water

"If I were to choose a single phrase to characterize the first century of modern oceanography, it would be a century of undersampling."

Walter Munk, Woods Hole Oceanographic Institute, 2000













Three Pillars of ESA Earth Observation



THE ESA EARTH OBSERVATION PROGRAMME

esa.int



European Space Agency

Examith a long-term operational perspectiv





Goal is a hierarchical system that integrates data and models (and can also be used to design observing systems)



Optimized sampling for multi-scale dynamics





Figure 1. Future role of wave models as an essential coupling component for ocean-atmosphere-carbon-cycle modets developed in the context of the World Climate and Global Change programs.







Sea-spray aerosol particles enriched in organic material are possibly generated when the air-sea interface is bursting



GRSAT

Ocean remote sensing: a privileged view

- Spatially detailed
 - Spatial resolution from meters to Kms
 - A synoptic picture that is 100 km 10 000 km wide
- Regularly repeated
 - Revisit intervals between 30 min. and 35 days
 - Continuously repeated over years to decades
- Global coverage
 - Satellites see the parts where ships rarely go
 - Single-sensor consistency no intercalibration uncertainties
- Measures parameters that cannot be observed in situ
 - Surface roughness at short length scales (2-50 cm)
 - Surface slope (a few cm over 100s of kilometres)



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THE ELECTRO MAGNETIC SPECTRUM Wavelength (metres) Visible Gamma Ray Radio Microwave Infrared Ultraviolet X-Ray 10-10 10-2 10-5 10-8 10-12 103 10-6 \mathcal{M} Frequency (Hz) 1012 1015 108 1016 1020 1018 104



Band name	Р	L	S	C	x I	K _u K _a	Q V W	
	0.39	1.55	4.	2 5.75	5 10.9	22 36	6 46 56	
Frequency	0.3 GHz	1.0	3.0		10	30	10	00 GHz
Wavelength	100 cm	30	10	(3.0	1.0	0	.3 cm

Table 2.1.	Band letter designations used in microwave remote
sensing.	9

Band	Frequency (GHz)	Wavelength			
Р	0.225-0.390	76.9–133 cm			
L	0.390-1.55	19.35–76.9 cm			
S	1.55-4.20	7.14–19.35 cm			
С	4.20-5.75	5.22–7.14 cm			
Х	5.75-10.9	2.75–5.22 cm			
K _u	10.9-22.0	1.36–2.75 cm			
Ka	22.0 36.0	8.33-13.6 mm			
Q	36.0 46.0	6.52-8.33 mm			
V	46.0 56.0	5.36-6.52 mm			
W	56.0 100	3.0 5.36 mm			



Satellite orbits

•Geostationary sensors typically offer a revisit interval of less than 30 min and spatial resolution of 1 to 5 km.

•The polar orbiting sensor cover the whole Earth in a single day if it is the swath at least 2700 km.

• Each point on the Earth surface is viewed once from descending track and once from ascending track.



Ground track of a typical near-polar, low-Earth orbit, showing all the descending passes for one day and one ascending pass (dashed).





Schematic illustrating the different remote-sensing methods and classes of sensors used in satellite oceanography, along with their applications (from Robinson, 2004).



Foremost, to understand the satellite sensor initial capability (sensor physics and spatio-temporal sampling) ->A consistent approach (T. Elfouhaily, 1997)











Signatures of 3 co-evolving 2015 major Hurricanes from 22 Aug to 9 Sep in the East and Central tropical Pacific as seen from SMOS, SMAP and AMSR-2 observations (beyond others)





Satellite instruments generally measure 2D surface expressions of 4D structures





Sub-mesoscale (10 km eddies) and high resolution radar sea surface roughness variations















Sensor Geometry: the SAR case





Sensor Geometry: the SAR case

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Cornwall, UK: wind gusts from land to sea from 52A sunglitter.

Oceanic and atmospheric process fingerprints on the sea surface





Satellite synthetic aperture radar (SAR)

light = more short waves

- ~ stronger wind forcing
- ~ air-sea heat, KE, momentum, gas exchange

dark = weak wind or surfactant



10 km

February 19, 2015 Image: MPC Sentine-1 portal wind

Corsica














Schematic illustration of the Langmuir circulation (first described by Langmuir, 1938). The separation scale of the convergence zones are typically 10-100 m





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wind

Corsica

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Typical Atmosphere Space-Time scales





Typical Ocean Space-Time Scales









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wind

Corsica

10 km

February 19, 2015 Image: MPC Sentine-1 portal









<u>Intense deformation field at oceanic front</u> <u>inferred from directional sea surface roughness</u> <u>observations</u>







Sea Surface Roughness changes: interpretation framework

$$\frac{\partial N(\mathbf{k})}{\partial t} + \left(c_{gi} + u_i\right)\frac{\partial N(\mathbf{k})}{\partial x_i} - k_j\frac{\partial u_j}{\partial x_i}\frac{\partial N(\mathbf{k})}{\partial k_i} = Q(\mathbf{k})/\omega \quad Q(\mathbf{k}) = \beta_\nu(\mathbf{k})\omega E(\mathbf{k}) - D(\mathbf{k}) - Q^{nl}(\mathbf{k}) + Q^{wb}(\mathbf{k})$$



$$\frac{\partial \tilde{N}(\mathbf{k})}{\partial t} + c_{gi} \frac{\partial \tilde{N}(\mathbf{k})}{\partial x_i}$$

= $\omega^2 k^{-5} \left[\omega^{-1} m_k^{ij} u_{i,j} B_0 - \tilde{B}/\tau + \tilde{\beta} B_0 + \tilde{I}_{sw} \right]$

$$m_k^{ij} = k_j \partial \ln N_0 / \partial k_i$$



M. SPOONER, SUR LA LUMIÈRE DES ONDES DE LA MER. 331

LETTRE XX.

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L'équation en question est:

$$\frac{a^2}{a^2 + x^2 + y^2} + \frac{2 \cos Z a}{\sqrt{a^2 + x^2 + y^2}} + \cos^2 Z = \frac{2 \cos^2 J + 3 \cos^2 J \cdot a \cdot \cos Z}{\sqrt{a^2 + x^2 + y^2}} - \frac{2 \cos J \sin Z \cdot x}{\sqrt{a^2 + x^2 + y^2}}.$$

Par la quatrième observation.

$$A^{*} = \frac{.0000013 + .0005593 + .0585262}{2 + .0005593 - 1.9281164} \cdot .0724429$$
De-là, log. $A = 1.05574725 = \log. \cos in. de 25^{\circ} 26'.$





JOURNAL OF THE OPTICAL SOCIETY OF AMERICA

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Measurement of the Roughness of the Sea Surface from Photographs of the Sun's Glitter

CHARLES COX AND WALTER MUNK Scripps Institution of Oceanography,* La Jolla, California (Received April 28, 1954)



FIG. 1. Glitter patterns photographed by aerial camera pointing vertically downward at solar elevation of $\phi = 70^{\circ}$. The superimposed grids consist of lines of constant slope azimuth α (radial) drawn for every 30°, and of constant tilt β (closed) for every 5°. Grids have been translated and rotated to allow for roll, pitch, and yaw of plane. Shadow of plane can barely be seen along $\alpha = 180^{\circ}$ within white cross. White arrow shows wind direction. Left: water surface covered by natural slick, wind 1.8 m sec⁻¹, rms tilt $\sigma = 0.0022$. Right: clean surface, wind 8.6 m sec⁻¹, $\sigma = 0.045$. The vessel Reverie is within white circle.



Cox and Munk (1956)





Fig. 13. Mean square slope components and their sum as functions of the wind speed W measured 41 ft. above sea level. The plot includes all analyzed data for clean sea surfaces (open circles) and slick surfaces (solid circles). Continuous lines are regression lines for clean surfaces; dashed lines for slick surfaces.

Paul Desmond Scully-Power NASA's astronaut



Gulf Stream roughness changes









Miso Turbulence (Mediterranean Sea) : pearl-necklace evidences













Total ocean surface current monitoring



New Era - Nanosatellites - CubeSat

A CubeSat is a type of <u>miniaturized satellite</u> for <u>space</u> <u>research</u> that usually has a volume of exactly 10 cm cube, and mass of no more than 1.33 kilograms.



Big Data & AI/Data Science

Data



Big data infrastructure

all a state



 \cap

n n



DL/AI







Numerous questions and challenges

Some of <u>the Living Planet Challenges</u> to better assess the existing pressures on the marine environment (e.g. overfishing, pollution, habitat destruction, ...) potentially leading to increased risks to global food security, economic prosperity, ...

- Evolution of coastal ocean systems including the interactions with land in response to natural and human-induced environmental perturbations
- Mesoscale and submesoscale circulation and the role of the vertical ocean pump and its impact on energy transport and biogeochemical cycles
- Response of the marine ecosystem and associated ecosystem services to natural and anthropogenic changes,
- Physical and biogeochemical air/sea interaction processes on different spatio-temporal scales and their fundamental role in weather and climate
- Sea level changes from global to coastal scales and from days (e.g. storm surges) to centuries (e.g. climate change)



ENVISAT MERIS Southern Ocean Bloom 13/01/2012



Numerous questions and challenges :

How can we map the distribution of marine plastic Debris?
Has the Agulhas current strengthened in the last 5 years?
Is the surface circulation of the Black Sea and in the Mediterranean Sea stable?
How is the Arctic Ocean changing ?
How is marine biodiversity changing, locally, regionally, globally ?
What is the extent of ocean acidification ?

Are western boundary currents changing, the Gulf Stream ?

How can ship routing be optimised ?

Why and where is regional sea level changing?

How are our coastal regions changing?

How can we map estuary systems from space?

Most observations are not yet sufficient

explored and used, limited to 'low-hanging fruits', and necessity to better optimize analysis to reveal multi-scale dynamics

- Synergy between high and medium resolution observations to reveal mean states and trends, near-surface ocean-atmosphere dynamics, local and non-local interactions, convergence/divergence surface fronts and numerous roughness contrasts
- Atmospheric and Oceanic observations generally produce high quality data, but it is often too sparse (many gaps where information is missing, and/or often too local in both space and time)
- How can we use observed data in combination with the physical knowledge of stochastic processes in nonlinear dynamical systems to estimate and model those effects on the variability of computationally resolvable scales of motion that are caused by the small, rapid, unresolvable scales of fluid motion that upscaling in data assimilation leaves out?



J. Tournadre LOS Ifremer

-150





Climate models are too coarse to resolve clouds



Global model: ~100 km resolution



Cloud scales: ~10-100 m



SST



MSG/SEVIRI (10km, 3 heures)



SAF O&SI NAR pour AVHRR17 (2km, 2 passes/jour)

1 - XI - XI





Avhrr 18



Avhrr 17



ENVISAT/AATSR (1 km, 14-15 orbites/jour)



SAF O&SI NAR18 pour AVHRR17 (2km, 2 passes/jour)

13/01/2008 8.5 12.0 15.5 Sea Surface Temperature (°C)



Multi-satellite product

Sea surface temperature

High resolution daily product 2006-present, 2 km resolution





lfremer

projet ESA Medspiration


Global SST



Global reanalysis 2006-present at 10 km resolution





High-resolution satellite ocean sensing

From low-resolution

to high-resolution



Low-resolution: ~25km (0.25°) eg, Altimetry (SSH), Radiometer (SST



Key issue

How to deliver daily high-resolution geophysical field at regional/global scale from the the irregular space-time sampling of high-resolution sensors

<u>Emulating the OCEAN at high-resoluton From</u> multi-scale/multi-sensor databases

HR observations are irregularly sampled in space and time. But 1) low resolution observations are generally available and 2) we should learn a lot from past joint HR/LR observations.



Key objective: learning new multi-scale/multi-modal representations of ocean dynamics from multi-sensor remote sensing archives



Multi-scale/multi-modal representations of ocean dynamics from remote sensing archives Wind fields:



Learning ECMWF-to-SAR transfer functions for HR wind field emulation



Multi-scale/multi-modal representations of ocean dynamics from remote sensing archives



SQG-likeassumption:localrelationshipsbetweenlocalpatchesandseasurfacecurrentsLearningfromSST-SSHobservations

Application to AMSR/SST-AVISO/SSH



Towards high-resolution sea surface currents from a joint SST-SSH analysis



SST - Modis(L2P)

SST - AMSRE(L3)





Characterizing the *submesoscale* - Spectral approach



Tracer mixing depends on KE spectrum

 $E(k) \sim k^{-\alpha}$ a>3 : spectrally nonlocal: stirring set by large scales a<3 : spectrally local: stirring set by local scales

- Simple proxy model: surface quasigeostrophic turbulence
- Dynamics driven by surface density anomalies (zero interior PV)





Stirring and mixing : interplay and scale interactions











06-May-2010 17:00 modis aqua











Lagrangian advection to dynamically interpolate largescale tracer (sea surface temperature field, left) onto a high-resolution product (right). Particle trajectories computed using altimetry-derived velocities (AVISO, weekly 1/3°) with 3 hours time steps





Stirring and mixing : interplay and scale









Observed data in combination with the physical knowledge of stochastic processes in nonlinear dynamical systems









06-May-2010 17:00 modis aqua











April 20

Application to Oil Spills Detection





The blended satellite products allow to estimate the impact of surface currents on the biogeochemical transport, on the dispersion of pollutants and oil spills



Forecast of oil spill dispersion in the Gulf of Mexico on 25 june 2010: red and blue show regions of strong oil dispersion within 3 days. This diagnosis, based on altimetric data, compared well with what was observed (Mezic et al, Science, 2010).

However these satellite datasets (altimetric and microwave data) cannot capture ocean dynamics at scales smaller than 100 km because of the resolution (or/and noise level).

LETTERAltimetry for ecology : the invisible landscape

doi:10.1038/nature10082

Tracking apex marine predator movements in a dynamic ocean

B. A. Block¹, I. D. Jonsen², S. J. Jorgensen¹, A. J. Winship², S. A. Shaffer³, S. J. Bograd⁴, E. L. Hazen⁴, D. G. Foley⁴, G. A. Breed^{2,5}, A.-L. Harrison⁵, J. E. Ganong¹, A. Swithenbank¹, M. Castleton¹, H. Dewar⁶, B. R. Mate⁷, G. L. Shillinger¹, K. M. Schaefer⁸, S. R. Benson⁹, M. J. Weise⁵, R. W. Henry⁵ & D. P. Costa⁵





NATURE | VOL 475 | 7 JULY 2011

In their displacements, top predators encounter environmental heterogeneity at multiple scales.

Until now, observations where sparse, and matched large-scale current information was enough

Eastern Pacific Freshpool & 3D monitoring of the pool



In situ analyses (depth)



Sea Surface Salinity from Space

Ionosphere



Sea Surface Temperature





Electromagnetic Models





Sea Surface Salinity







Sea State







SSS Monthly Composite Jan 2010-0.5°x0.5° SSS Monthly Composite Feb 2010-0.5°x0.5° SSS Monthly Composite Mar 2010-0.5°x0.5°





SSS Monthly Composite Jul 2010-0.5°x0.5° SSS Monthly Composite Aug 2010-0.5°x0.5° SSS Monthly Composite Sep 2010-0.5°x0.5°



SMOS SSS (color)+ currents (vector) from 03/03 to 17/03 2012











Lagrangian Optical-Physical properties









SSS Averaged from Jun 04 through Jun 14







FIG.2 The number of 1950 through 2010 "best track" TC per one degree square (smoothed by a 3° x 3° block average) (a) that evolves as Cat 4-5 somewhere along their path and (b) that intensified locally to Cat 4-5. The black curve is showing the historical extent of the Amazon-Orinoco river plume during the hurricane peak season (August to October).





Passive Microwave measurements



Semi-empirical model

-Self-similar breaker-scale distribution

-Foam coverage and thickness \rightarrow wavelength sensitivity





#IrmaHurricane2017





Ocean Surface Wind : C-Band Synthetic Aperture Radar

Signal	Intensity of backscatter signal (NRCS)
Resolution	1 km
Assets	Dual polarization: -Co-pol = less noise, sensitive to wind direction -Cross-pol = no saturation at high winds High resolution
Limitations	Not a large coverage, reduced opportunity to get acquisitions over TCs Impacts of rain and waves exist





Ocean Surface Wind : C-Band Synthetic Aperture Radar

Thanks to its high resolution C-Band SAR can be used to estimate Tropical Cyclones parameters, including those related to the inner core: radius of maximum wind speed, maximum wind speed, the inner core enthropy This is complementary to radiometer large coverage

> Ocean surface wind speed [knts] - Omnidirectionnal -






Upper ocean responses to extreme wind forcing by tropical cyclones remains a central problem in physical oceanography

Parameterization of wind forcing at high-wind speeds is still a matter of debate and active research

To accurately model air-sea coupled processes, it is of central importance to quantify how efficiently surface winds and waves, within storm cores, increase the momentum of surface currents

Energy transfer to upper currents mostly occurs in front of the extremes and is nearly balance with the increase in kinetic energy of the upper ocean currents









Far from the coasts, Extreme Events are opportunities of high scientific values to investigate how natural processes at their peaks can transfer energy and matter within and across boundaries, and to identify the mechanisms involved and their rates, jointly with their local and/or long term impacts



R_{max}

(a)



Simplified TC Wake Model

Anomalies

Anomalies

Surface signature wakes of Igor

SAT





Surface altimeter SSH wakes of Igor





Surface SSH wake and persistency of Igor





ARGO Floats









Surface area~ 89000 km²> Lake Superior, the world largest freshwater lake: a transfer of 1 GTo of Salt in 5 days



SMOS microwave satellite-derived SSS composite images of the Amazon plume region revealing the SSS conditions (a) before and (b) after the passing of Hurricane Igor, a category 5 hurricane that attained wind speeds of 136 knots in September 2010. Color-coded circles mark the successive hurricane eye positions and maximum 1-min sustained wind speed values in knots. Seven days of data centered on (a) 10 Sep 2010 and (b) 22 Sep 2010 have been averaged to construct the SSS images, which are smoothed by a 1° x 1° block average.



<u>DIX question</u>is....

Are storms more numerous and intensifying with climate change?





Seismic noise (50 years)



Buoys (30 years)







Identifying swell field source

139

- Linear theory of swell propagation:
 - in open ocean, far from islands
 - swell propagates at group speed, Cg=gT $_p/4\pi$
 - along great circles of direction Θ_{p}

 \rightarrow compensate for sparse and track-based sampling of the swell partitions.

- Refocusing of the swell partitions
 - → converge in space and time to regions systematically (96% collocations) coinciding with Storms events

GRSAT

Wind speed: 31.8 m/s Swath date: 09/07/2004 06:25

-40° -45° -50° -55° -60° -65° -70° -75°

150° 155° 160° 165° 170° 175° 180°-175°

30

25

20

- 15

10

5

Example of seismic - SAR synergy

Signatures of 3 co-evolving 2015 major Hurricanes from 22 Aug to 9 Sep in the East and Central tropical Pacific as seen from SMOS, SMAP and AMSR-2 observations (beyond others)

Jimena : wave generationx

- Example of retro-propagated Sentinel-1 A Swell Measurements. Data acquired the 2015 Sept 8 16:40 to 16:46 UTC
- 3 tracks corresponding to the 3 hurricanes Kilo, Ignacio and Jimena (from left to right) are overplotted. Color code is time.

- Example of retro-propagated Sentinel-1 A Swell Measurements. Data acquired the 2015 Sept 8 16:40 to 16:46 UTC
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- Refocalisation area is found along the Jimena track the 6th of September. On the right hand side of the track.

Trapped-fetch waves – distinctive feature of TC wave field

Kilo : wave generation

GRSAT

Kilo : wave generation

Main message ...

- Today ideal instrument ... (wide-swath, high-resolution, topography, roughness, Doppler, emissivity, reflectance, ...) = the <u>combined</u> use of observations, including in situ measurements
- Very (too) large number of spatio-temporal scales under local and non-local interactions
- Improved technologies (instruments, resolution, computer capabilities, storage, dissemination) all contribute to improved <u>combined</u> analysis
- Theoretical frameworks and numerical simulations can be used to assess the <u>causes</u> and <u>contexts</u> of the different observations (including sensor physics, observability conditions and instrument capabilities), to refine dynamical/statistical gap filling methods
- New challenges, new altimeter instruments (SARAL, Sentinel-3, SWOT, ..., CubeSat opportunities), possible new high-resolution microwave instruments (10-20 km), Doppler measurements to infer sea surface currents (SAR and/or RAR-SKIM or DopplerScat), and combined roughness contrasts as local quantitative proxies to trace strong surface gradient areas

Opening the Pandora's box?

Archiving data leads to very large heterogeneous and multimodal databases

Data assimilation is growing in response to the growth of data collected, but (personal opinion) tremendous amounts of information still remain hidden in data archives.

Knowledge trees and complex algorithms are essential to avoid the Google's principle, i.e. pertinence = popularity

Research efforts to be concerned with the definition of adequate exploratory processes to detect relevant patterns in large, heterogeneous, multidimensional observation data sets with different resolutions to better approach complex spatial and/or temporal dynamics of the ocean system.

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